Modeling Investment Decisions in Gas Pipeline Networks: A Game-theoretic Mixed Complementarity Problem Formulation and its Application to the Chinese Gas Market

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Abstract:

Rapidly growing natural gas demand in China has formed a precondition to investigate the potential of several pipelines from Russia and other CIS countries, which could possibly head their flows to the Chinese natural gas market. The variety of proposed projects and the difference in their characteristics results in competition and, therefore, in order to assess the projects' economic perspectives a game-theoretic approach is used. A two-step procedure for the problem's game-theoretic formulation is proposed.

In the instantaneous supply game the players have the possibility to supply natural gas to the market. Each player determines his or her supply levels in such a way that profits are maximized, taking into account the supplies of all other players.

In the game of timing the players have to decide when to construct their pipelines. Due to the growing demand for natural gas, the time of entering the market plays an important role. The players seek to maximize the net present value of their investment by choosing the pipelines' start of commercial operation from a discrete set of years. The equilibria obtained from the supply game are used to calculate the payoff matrices for each pipeline in the timing game.

Both games are formulated and implemented as mixed complementarity problems which allows the analysis of the competition of up to 5 pipeline projects simultaneously. Results for the Chinese natural gas market are presented and discussed with an emphasis on sensitivity analysis.

1 Introduction

Of all fossil energy carriers, natural gas is expected to show the strongest growth in demand over the next decades. In its World Energy Outlook 2002 the International Energy Agency (IEA) forecasts an average annual growth of world natural gas consumption by 2.4% between 2000 and 2030. The estimates for East Asia are significantly higher and show a demand growth of 3.7% per annum in the same time period [IEA 2002b]. The spread of growth rates among the region's countries is considerable, with China showing the highest demand growth of annually 5.5%. Other forecasts even indicate growth rates of up to 8.3% for China between 1990 and 2020 [APERC 2002], resulting in a considerably higher demand in 2020. For an overview of different projections of China's natural gas demand see [IEA 2002a].

This rapidly evolving natural gas demand in China, which is also expected to result in high import requirements in the future, has led to planning activities for several pipelines from Russia and other CIS countries to the Chinese market. Economic and technical data of the pipeline projects show significant differences which suggests that some have better chances to compete successfully on the market than others. Both the expected growth of the demand on the northeast Asian gas markets and the numerous competing projected pipelines form a precondition for a game-theoretic analysis of the situation. Since many of the pipelines are proposed by different companies or consortia, non-cooperative game theory provides a possible framework for this analysis [Nash 1950].

Two factors mainly determine the behavior of the pipeline projects' managements, (i) with growing demand the expected profits increase, so it might be advisable to postpone the market entry for a while and (ii) once other pipelines are already active on the market, due to competition among the suppliers their profits are expected to decrease. The time of entering the market is therefore of particular importance for the projects to be profitable. Thus, there is a game situation where the key parameters are times, at which the participants enter the market. This concept has been formalized in the context of collaborative studies undertaken by IIASA's Environmentally Compatible Energy Strategies (ECS) and Dynamic Systems (DYN) Programs and a network of Russian institutions. In [Klaassen et al. 2004] the timing game between two competitors was described, its mathematical structure was analyzed in detail, followed by illustrative examples on the Turkish and Chinese natural gas markets. Other applications of the concept oriented to Turkey's natural gas market can be found in [Klaassen et al. 2003, Golovina et al. 2002]. Results have also been presented on a number of conferences [Klaassen et al. 2002, Minullin et al. 2002, Minullin et al. 2001].

In this paper we follow the basic idea presented in [Klaassen et al. 2004]. However, since our main focus is the model's applicability, the formalization and implementation of the game situation differ from the original concept in [Klaassen et al. 2004]. We define discrete commercialization times as strategies, which allow us to compute Nash equilibria in practical applications for up to 5 competing projects. The problem is implemented in GAMS [Brooke et al. 1998] as a mixed complementarity problem (MCP) and the solvers PATH [Dirkse et al. 1995, Ferris et al. NA] and MILES [Rutherford 1993] are used for locating Nash equilibria.

The paper is structured as follows: Section 2 describes the game-theoretic modeling approach that is applied in the analysis. In section 3 details on the model implementation as a mixed complementarity problem (MCP) are presented. A brief description of the software that has been developed for practical applications follows. Section 4 is dedicated to the treatment of various sources of uncertainties within the model. In section 5 a short overview over the expected development of the natural gas markets in China and the trans-national pipelines that have been proposed to satisfy the growing demand for gas is given. The results of model calculations for a number of pipeline projects to China are presented and discussed in section 6. The paper ends with an outlook on further possible development activities of the model in section 7.

2 Model Description

The modeling of the pipeline game can be divided into two parts, the so-called *instantaneous supply game* and the *game of timing* [Klaassen et al. 2004, Klaassen et al. 2003].

In the *instantaneous supply game* a number of players have the possibility to supply natural gas to a market. Each player then chooses his or her supply level to the market in such a way that his or her profits are maximized, taking into account the supplies of all other players. Due to the pipelines' limited capacities, the competition can be formulated as a Nash-Cournot game [Cournot 1838] with additional constraints.

In the *game of timing* the projects' commercialization times are the strategic variables. The equilibria obtained from the instantaneous supply game are used to calculate the net present value of a pipeline project. Obviously this value does not only depend on the commercialization time of a single player, but also on the commercialization times of all other players. Choosing discrete time periods as strategies, this can formulated as a game in normal (or strategic) form. For this class of games it can be shown that there always exists a Nash equilibrium in mixed strategies (see e.g. [Fudenberg et al. 1991]).

2.1 Terminology and Notation

In this paper we use the terms project, pipeline and player interchangeably when referring to the pipeline projects either in a game-theoretic or economic context. The term *commercialization time*, introduced above, will be used with respect to the point of time, representing completion of the construction and start of commercial operation of a pipeline.

All pipeline projects are characterized by their economic and technical parameters, such as investment cost, fixed and variable operation and maintenance costs, capacity and technical lifetime. In addition to the parameters characterizing the projects, assumptions on the development of the markets have to be made. In the model the behavior of the demand side is described by an inverse demand function (price formation function). This description corresponds to a liberalized (spot) market for natural gas. Table 1 provides an overview of all parameters that are used in the model's mathematical formulation.

parameter	description	units[1]
p_0	natural gas price at given demand d_0	[$/1000cm]
e_p	absolute value of price elasticity at demand d_0	[-]
r_i	discount rate for player i	[-]
M_i	pipeline capacity for player i	[bcm/a]
c_i^{ext}	specific gas extraction costs	[$/1000cm]
c_i^{var}	specific variable transport costs	[$/1000cm]
c_i^{fix}	specific fixed annual costs	[mill.$/(bcm/a)]
c_i^{inv}	total investment costs	[mill.$]
c_i^{ann}	annuity of investment costs	[mill.$]
t_i^{tech}	technical lifetime of pipeline i	[a]
t_i^{eco}	economic lifetime of pipeline i	[a]
$p_{\max}$	exogenous upper price limit	[$/1000cm]

Table 1: Parameters used in the formalization of the games.

The formulations of both the instantaneous supply game and the game of timing are universal for any number of players. Thus the number of players n that participate in the

[1] cm = cubic meter (m^3), bcm = billion cubic meter (10^9 m^3)

game is in principle arbitrary. However, when analyzing the situation on a particular market, for numerical reasons we restrict ourselves to $n \leq 5$.

As it is common practice in the game-theoretic literature, if the strategy space of player i is denoted by S^i and $s_i \in S^i$ are the corresponding strategies, we denote by s_{-i} the set of strategies of all players except those of player i

$$s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n) \ .$$

Accordingly, the Cartesian product of strategy spaces of all players i apart from that of player i is denoted by

$$S^{-i} = S^1 \times \ldots \times S^{i-1} \times S^{i+1} \times \ldots \times S^n \, .$$

2.2 Instantaneous Supply Game

As briefly mentioned in the previous paragraphs, the instantaneous supply game models the optimal behavior of players which have the possibility to supply natural gas to a market. Hence, each player will choose his or her supply level in such a way that profits are maximized, taking into account the supplies of all other players. We assume perfect information, i.e. each player knows all parameters of the other projects and in turn they know that he knows and so on.

2.2.1 Game Formalization

Let there be n profit maximizing players that consider building a pipeline to deliver natural gas to a market. Let y_i be the positive supply levels which are also limited by the maximum pipeline capacities M_i. Hence the amount of natural gas delivered by player i to the market is constrained by the following restrictions

$$y_i \geq 0 \quad \text{and} \tag{1}$$

$$y_i \leq M_i\,. \tag{2}$$

Let $p(y)$ denote the inverse demand function, where y is the total amount of gas on the market

$$y = y_{\text{ext}} + \sum_i y_i\,. \tag{3}$$

Here y_{ext} is the amount of gas supplied by numerous producers that are not considered explicitly in the game. As a first step, we introduce a linear inverse demand function, having two independent parameters. A discussion of different forms of inverse demand functions follows in section 4.2. We choose to parameterize the function by the price p_0 and the price elasticity's absolute values e_p at a given demand d_0. Under the assumption that market is in equilibrium, i.e. supply equals demand, the linear inverse demand function's parameterization in terms of p_0 and e_p looks as follows

$$p(y) = \frac{p_0}{e_p}\left(1 + e_p - \frac{y}{d_0}\right), \tag{4}$$

where y is the above defined total supply of natural gas to the market. The profit of player i is then given by

$$\pi_i(y_i, y_{-i}) = \left[p(y) - c_i^{\text{ext}} - c_i^{\text{var}}\right] y_i - c_i^{\text{fix}} \cdot M_i - c_i^{\text{ann}}\,, \tag{5}$$

where we use the annuity of investment costs, which is defined as

$$c_i^{\text{ann}} = c_i^{\text{inv}} \frac{(1+r_i)^{t_i^{\text{eco}}} \cdot r_i}{(1+r_i)^{t_i^{\text{eco}}} - 1}\,. \tag{6}$$

In this formula r_i is the discount rate and t_i^{eco} the economic lifetime of project i. Although the model is built in such a way that there are different values possible for the players, we usually work with the same values for all players and only consider impacts of deviations from this practice in the sensitivity analysis.

2.2.2 Price Constraint

Natural gas, delivered by pipeline (PNG) has to compete with alternative fuels on the energy markets. Apparently liquefied natural gas (LNG) is such an alternative which can substitute PNG directly. Supplying the same final product and thus competing on the same markets, LNG terminal projects can be modeled in exactly the same way as pipeline projects. However, in the considered example we assume that the spot market price p_{lng} in the regional LNG market will serve as an upper bound for the endogenously calculated price (4). Furthermore, the possibility to switch to other energy carriers (e.g. coal, oil, renewables) will act as an upper bound on the price of gas. Since such alternatives are not explicitly present in the model, the only possibility to include them effectively is via the simple condition

$$p(y) \leq p_{\mathrm{max}} \,. \tag{7}$$

Obviously the upper price limit p_{max} is an exogenously defined quantity. An upper bound on the price $p(y)$ can be directly translated into a lower bound y_{min} for the supplied quantity $y(p)$ by applying eq. (4)

$$y_{\mathrm{min}} = d_0 \cdot \left[1 + e_p \left(1 - \frac{p_{\mathrm{max}}}{p_0}\right)\right] \,. \tag{8}$$

At this point some care is needed, since in the case of a sufficiently low price limit p_{max} the problem becomes infeasible. If $\sum_i M_i < y_{\mathrm{min}}$, eq. (8) is in contradiction with the pipeline capacity constraint (2). In such a situation, the model endogenous price $p(y)$ has to be fixed to the level of p_{max} and the game theoretic problem is reduced to a market with perfect competition, i.e. the players are price takers. Equation (5) simplifies to

$$\pi_i(y_i) = \left[p_{\mathrm{max}} - c_i^{\mathrm{ext}} - c_i^{\mathrm{var}}\right] y_i - c_i^{\mathrm{fix}} \cdot M_i - c_i^{\mathrm{ann}} \tag{9}$$

and each player's choice is independent of the other players' strategies.

2.2.3 Optimization with Inequality Constraints

However, if eq. (8) is not in contradiction with (2), it is possible to include (7) as a constraint into the model and thus force the players to supply as much natural gas as necessary to ensure that the endogenously determined price (4) does not exceed the exogenous price limit $p_{\max}$.

To define the maximization problem under the (inequality) constraints (2) and (7), we have to introduce Lagrange parameters $\lambda_i^{\max}$ and λ_p. Hence we obtain the problem's Lagrange function for player i

$$L_i(y_i, y_{-i}) = \pi_i(y_i, y_{-i}) - \lambda_i^{\max}(y_i - M_i) - \lambda_p (p(y) - p_{\max}) \ .$$

In this game, the Nash-Cournot equilibrium is obtained by solving the quadratic system of equations that results from the Kuhn-Tucker conditions of the constrained (inequality) maximization problem for all players. The conditions for player i are defined to be

$$\begin{array}{llll} & & & \dfrac{\partial L_i(y_i, y_{-i})}{\partial y_i} = 0\,, \\ \lambda_i^{\max} \geq 0 & , \quad y_i \leq M_i & \text{and} & \lambda_i^{\max}(y_i - M_i) = 0 \\ \lambda_p \geq 0 & , \quad p(y) \leq p_{\max} & \text{and} & \lambda_p (p(y) - p_{\max}) = 0\,. \end{array} \tag{10}$$

The latter two rows of equations are called complementary slackness conditions, because at most one of them is slack, i.e. not an equality.

For the simple price formation mechanism (4) the partial derivative of the Lagrange function with respect to the supply of player i is

$$\frac{\partial L_i(y_i, y_{-i})}{\partial y_i} = \frac{p_0}{e_p}\left[1 + e_p - \frac{1}{d_0}\left(y_{\text{ext}} + \sum_j y_j + y_i\right)\right] - (c_i^{\text{ext}} + c_i^{\text{var}}) - \lambda_i^{\max} - \lambda_p \frac{p_0}{e_p \cdot d_0}\,. \tag{11}$$

As can be seen easily from eq. (5), in the case of the linear inverse demand (4), $\pi_i(y_i, y_{-i})$ is concave for every i. Furthermore the restrictions (2) and (7) are linear. Hence the Kuhn-Tucker conditions (10) are both necessary and sufficient for a global optimum [Kuhn et al. 1951].

However, by assuming a different inverse demand function, e.g. an isoelastic one, this is not necessarily true any longer and the concavity of $\pi_i(y_i, y_{-i})$ has to be established explicitly (see section 4.2).

The value of the Lagrange multipliers at the problem's solution are equal to the rate of change in the maximal value of the objective function(s) as the constraint is relaxed. Hence $\lambda_i^{\max}$ and λ_p can be considered as the shadow prices of the corresponding restrictions.

2.3 Game of Timing

In contrast to the above described instantaneous supply game, the game of timing is modeled as a finite n-person game in normal form. This is achieved by allowing only discrete time steps as strategies, i.e. as possible commercialization years of the projects. Therefore its formulation is different from that of the above discussed Nash-Cournot game, although – as will be shown later – it can be treated within the same class of optimization problems.

2.3.1 Game Formalization

The model's time horizon is defined by the (discrete) set of years t,

$$t = \{t^0 \dots t^1\} \,. \tag{12}$$

Here t^0 corresponds to the first model year that is also used as a base year for discounting. The years of project commercialization t_i, which are the strategies of the players, form subsets of t. These subsets do not necessarily have to be the same for all players, due to possible constraints related to individual projects. For example one can imagine a situation in which one of the projects cannot enter the market prior to a certain year for technical or even administrative reasons. Thus we define the strategy set for player i to be

$$S^i = \{t_i^0 \dots t_i^1, e_i\} \quad \text{with } t_i^0 \geq t^0 \text{ and } t_i^1 \leq t^1 \,, \tag{13}$$

where e_i is an (optional) exit strategy, which corresponds to the decision not to build the pipeline at all. For the sake of a simplified notation, we will use the following unified syntax

$$S^i = \{s_i^1 \dots s_i^{m_i}\} \quad \text{with } m_i = (t_i^1 - t_i^0 + 1) + \delta_{ie} \,, \tag{14}$$

where $\delta_{ie} = 1$ if an exit strategy for player i is defined and 0 otherwise.

The project's net present value over the whole time horizon serves as objective function of each player. Therefore the payoff functions Π_i are defined as follows

$$\Pi_i(s_i, s_{-i}) = \sum_{t=t^0}^{t^1} (1 - r_i)^{t-t^0} \pi_i(y_i(t), y_{-i}(t), t) \cdot \Theta(t - t_i) \cdot \Theta(t_i + t_i^{\text{tech}} - t) \cdot (1 - \delta_{s_i e_i}) \,. \tag{15}$$

$\pi_i(y_i(t), y_{-i}(t), t)$ are the profit functions from the instantaneous supply game for a given time period t and $(1 - r_i)^{t-t^0}$ is the discount factor relative to the base year t^0. For a given combination of strategies s_i, s_{-i} the supply game's equilibrium solutions $y_i(t)$, $y_{-i}(t)$ are used to calculate the cash flow for all year t, corresponding to the particular market situation. The Θ- or unit-step-function is defined as

$$\Theta(t) = \begin{cases} 1 & , \quad t \geq 0 \\ 0 & , \quad t < 0 \end{cases} \,, \tag{16}$$

where the first Θ-function in (15) ensures that a player has entered the market before making profits, i.e. the strategy t_i corresponds to a year after the current time t in the sum. The second Θ-function guarantees that a pipeline is still within its technical lifetime. Here we interpret the technical lifetime as a period after construction during which the pipeline can operate without making further investments. In fact this time span is not fixed and in reality can be prolonged by improved maintenance activities. However, we exclude this possibility in this paper and use fixed values for the pipelines' lifetime.

The term containing the Kronecker delta $(1 - \delta_{s_i e_i})$ symbolizes that profit function equals zero in the case of choosing the exit strategy e_i, independent from the choice of the other players. Formally, the Kronecker delta is defined to be

$$\delta_{s_i s_j} = \begin{cases} 1 \quad , \quad s_i = s_j \\ 0 \quad , \quad s_i \neq s_j \end{cases} . \tag{17}$$

2.3.2 Example: 2-player game

To illustrate the calculation procedure for the payoff matrices $\Pi_i(s_i, s_{-i})$ we will have a look at a 2-player game with just two time periods $t = 0, 1$, two possible choices for the commercialization times $t_i = 0, 1$ for $i = 1, 2$ and no exit strategies. Apparently there are altogether four possible payoffs for each player in this situation, which are shown below.

$$\begin{aligned}
\Pi_i(t_1 = 0, t_2 = 0) &= (1 - r_i)^0 \cdot \pi_i(y_1(0), y_2(0), 0) &+ (1 - r_i)^1 \cdot \pi_i(y_1(1), y_2(1), 1) \\
\Pi_i(t_1 = 0, t_2 = 1) &= (1 - r_i)^0 \cdot \pi_i(y_1(0), 0, 0) &+ (1 - r_i)^1 \cdot \pi_i(y_1(1), y_2(1), 1) \\
\Pi_i(t_1 = 1, t_2 = 0) &= (1 - r_i)^0 \cdot \pi_i(0, y_2(0), 0) &+ (1 - r_i)^1 \cdot \pi_i(y_1(1), y_2(1), 1) \\
\Pi_i(t_1 = 1, t_2 = 1) &= (1 - r_i)^0 \cdot \pi_i(0, 0, 0) &+ (1 - r_i)^1 \cdot \pi_i(y_1(1), y_2(1), 1)
\end{aligned}$$

As already mentioned, the payoff functions $\pi_i(y_1(t), y_2(t), t)$ have to be chosen in such a way, that they represent the situation corresponding to the strategy combination $s_1 = t_1$, $s_2 = t_2$. For example if $t \geq t_1$ and $t < t_2$, the payoff function for only player 1 being present on the market has to be summed for this particular period t. For $t \geq t_1$ and $t \geq t_2$ $\pi_i(y_1(t), y_2(t), t)$ has to represent a situation where both players are participating in the instantaneous supply game at time t. A "0" as a supply level of a player indicates absence from the market.

As it was shown in [Klaassen et al. 2004] $\Pi_1(t_1 = 2, t_2 = 1)$ equals $\Pi_1(t_1 = 2, t_2 = 2)$ and $\Pi_2(t_1 = 1, t_2 = 2)$ equals $\Pi_2(t_1 = 2, t_2 = 2)$. This symmetry can be used to reduce the number of operations to calculate the payoff function matrices $\Pi_i(t_i, t_{-i})$ for the game of timing, which can become time consuming. For n players, each of which has m strategies, the number of assignments is $n \cdot m^n$ and can be reduced to $n \cdot \sum_{j=1}^{m} j^{n-1}$ by applying the generalized n-player symmetry.

2.3.3 Mixed Strategies

For the game of timing a Nash equilibrium in pure strategies might not exist. It can be shown though that finite n-player games in normal form always have a Nash equilibrium in mixed strategies (Nash's theorem [Nash 1951]). Although Nash equilibria in mixed strategies might appear to be of little help for the players, because the investment decision has to be made once and for all, the credible implementation of a mixed strategy (e.g. to toss a coin) can force the competitors to react differently, resulting in another Nash equilibrium. From the computational point of view mixed strategies have the advantage of introducing continuous variables. This allows us finally to formulate the game of timing as a mixed complementarity problem as will be shown in section 3. Furthermore, pure strategies are a special case of mixed strategies and thus can also be found in principle by the same algorithm.

Now we formally introduce mixed strategies and subsequently derive optimality conditions for the players. The set of mixed strategies for player i is defined to be

$$\Delta(S^i) = \left\{ q_i : S^i \to R^+ \mid \sum_{s_i \in S^i} q_i(s_i) = 1 \right\} , \tag{18}$$

where $q_i(s_i)$ is the probability with which player i chooses the pure strategy $s_i \in S^i$. If the players choose the n-tuple $q = (q_1, \ldots, q_n)$ of mixed strategy vectors, the expected payoff to player i is

$$\psi_i(q_i(s_i), q_{-i}(s_{-i})) = \sum_{\substack{(s_1,\ldots,s_n)\in \\ S^1 \times \ldots \times S^n}} \prod_{j=1}^{n} q_j(s_j) \cdot \Pi_i(s_1, \ldots, s_i, \ldots, s_n) \, . \tag{19}$$

2.3.4 Optimization with Equality Constraints

Similar to the instantaneous supply game we can now introduce a Lagrangean with a set of equality constraints, that ensure the sum of the probabilities to be unity $\sum_{s_i} q_i(s_i) = 1$ for each individual player i

$$L_i(q_i(s_i), q_{-i}(s_{-i})) = \psi_i(q_i, q_{-i}) - \lambda_i \left(\sum_{s_i \in S^i} q_i(s_i) - 1 \right) . \tag{20}$$

The derivative of L_i with respect to all $q_i(s_i)$, $s_i = s_i^1 \dots s_i^{m_i}$ provides the first-order conditions for this problem

$$\begin{aligned} 0 &= \frac{\partial L_i(q_i(s_i), q_{-i}(s_{-i}))}{\partial q_i(s_i)} \quad \text{and} \\ 0 &= \frac{\partial L_i(q_i(s_i), q_{-i}(s_{-i}))}{\partial \lambda_i} . \end{aligned} \tag{21}$$

By inserting (20) and (19) into conditions (21) explicitly, we obtain the following set of $(m_i^+ - m_i^- + 1) + 1$ equations for every player i

$$0 = \sum_{s_{-i} \in S^{-i}} \prod_{\substack{j=1 \\ j \neq i}}^{n} q_j(s_j) \cdot \Pi_i(s_i, s_{-i}) - \lambda_i \,, \tag{22}$$

$$0 = \sum_{s_i \in S^i} q_i(s_i) - 1 \,. \tag{23}$$

Having derived the first-order conditions for both parts of the game situation, we can proceed to their formulation and implementation as so-called mixed complementarity problems.

3 Mixed Complementarity Problem

In this section we will describe the formulation of both the supply game and the game of timing as complementarity problems. Their implementation in GAMS (General Algebraic Modeling System) as mixed complementarity problems (MCP) will also be discussed briefly.

3.1 Model Formulation as MCP

It is a well known fact that n person non-cooperative games can be formulated as variational inequality problems under certain conditions [Ferris et al. 1997]. For Nash-Cournot games and n player matrix games (generalization of bimatrix games) it is possible to find an implementation as complementarity problems. Using modeling languages such as AMPL or GAMS and the standard solvers that come along with them, these can be handled without having to implement the solution algorithms (e.g. [Ferris et al. 2000]).

3.1.1 Definition of MCP

The classical nonlinear complementarity problem (NCP), defined by the nonlinear function $F : \mathbb{R}^n \to \mathbb{R}^n$, is to find a $x \in \mathbb{R}^n$ such that

$$0 \leq x \perp F(x) \geq 0 \,. \tag{24}$$

Here the $\perp$ notation is used to signify that componentwise one of the two adjacent inequalities must be satisfied as an equality, i.e. in addition the condition $x^T F(x) = 0$ also holds. Equivalently, componentwise this can be stated as

$$x_i \geq 0 \,, \quad F_i(x) \geq 0 \,, \quad x_i F_i(x) = 0 \,, \quad i = 1, \ldots, n \,. \tag{25}$$

The Kuhn-Tucker conditions (10) and (11) of the instantaneous supply game are of similar form. To achieve the exact form of an NCP condition, eq. (11) has to be paired with the supply variables y_i. The meaning of the pairing is that either the supply variable is zero or, if $y_i > 0$, the corresponding condition to maximize profits has to hold. Thus it is straightforward to treat the supply game as a NCP.

A generalization of the NCP is the mixed complementarity problem (MCP) which allows the treatment of both inequalities and equalities. The latter have to be paired with unconstrained (free) variables. The MCP is also referred to as rectangular [Harker et al. 1990] or box-constrained [Ferris et al. 1997] variational inequality. In the case of m inequalities and $n - m$ equalities the MCP has the following form

$$\begin{aligned} &x_i \geq 0 \,, \quad F_i(x) \geq 0 \,, \quad x_i F_i(x) = 0 \,, \quad && i = 1, \ldots, m \\ &x_i \text{ free} \,, \quad F_i(x) = 0 \,, && i = m+1, \ldots, n \,. \end{aligned} \tag{26}$$

Comparing eqs. (22) and (23) with (26) their similar structure becomes apparent. As for the NCP, the pairing of variables and equations still has to be done for (22). Not being constrained by any restrictions, the Lagrange parameters λ_i have to be paired with equations (23). On the other hand the probabilities $q_i(s_i)$ apparently have to be positive. Following the same argument as above, the probabilities $q_i(s_i)$ thus have to be paired with eqs. (22).

In the following we recapitulate the formulation of both games in the MCP form, using the $\perp$ notation. Here, we also incorporate a peculiarity of GAMS which only allows to pair a variable with a lower bound with an $\geq$ equation and a variable with an upper bound with an $\leq$ equation [McCarl 2004, Chap. 38]. Since an equation can be written either way by simply exchanging the left and the right hand side, this is no restriction, but just has to be kept in mind when implementing the MCP in GAMS.

3.1.2 Instantaneous Supply Game

The profit condition for player i is expressed by

$$y_i \geq 0 \quad \perp \quad 0 \geq \frac{p_0}{e_p}\left[1 + e_p - \frac{1}{d_0}\left(y_{\text{ext}} + \sum_j y_j + y_i\right)\right] - (c_i^{\text{ext}} + c_i^{\text{var}}) - \lambda_i^{\max} - \lambda_p \frac{p_0}{e_p d_0}\,. \tag{27}$$

Due to the limited pipeline capacity we have an upper bound on the supplied quantity by each player.

$$\lambda_i^{\max} \geq 0 \quad \perp \quad M_i \geq y_i \tag{28}$$

The endogenously determined natural gas price can be calculated as the model is running, but it merely acts as a balance equation.

$$p \text{ free} \quad , \quad p = \frac{p_0}{e_p}\left[1 + e_p - \frac{1}{d_0}\left(y_{\text{ext}} + \sum_i y_i\right)\right] \tag{29}$$

In the case of a maximum price $p_{\max}$ that is lower than the endogenous market price p when all pipelines run at full capacity, it is assumed that the price equals the maximum price and the problem reduced to an optimization problem for each player. This situation corresponds to a market with perfect competition, the players are price takers and (27) is replaced by the much simpler condition

$$y_i \geq 0 \quad \perp \quad 0 \geq p_{\max}(t) - (c_i^{\text{ext}} + c_i^{\text{var}})\,. \tag{30}$$

If the capacities of all existing pipelines are large enough to supply quantities that do not let the endogenously calculated price exceed the exogenously given maximum price, the following equation applies.

$$\lambda_p \leq 0 \quad \perp \quad \frac{p_0}{e_p}\left[1 + e_p - \frac{1}{d_0}\left(y_{\text{ext}} + \sum_i y_i\right)\right] \leq p_{\max} \tag{31}$$

3.1.3 Game of Timing

Algorithms for finding Nash equilibria in 2-player bimatrix games have been established long ago [Lemke et al. 1964]. A generalization to n-player matrix games is also available (e.g. [Ferris et al. 1997] or [McKelvey et al. 1996]). Although it is straightforward to rewrite the conditions for finding a Nash equilibrium in mixed strategies as a complementarity problem, problems might arise in the computational treatment [McKelvey et al. 1996].

The MCP formulation of the players' profit condition reads

$$q_i(s_i) \geq 0 \quad \perp \quad 0 \geq \sum_{s_{-i} \in S^{-i}} \prod_{\substack{j=1 \\ j \neq i}}^{n} q_j(s_j) \cdot \Pi_i(s_1, \ldots, s_i, \ldots, s_n) - \lambda_i \quad . \tag{32}$$

As a binding constraint the sum over the probabilities $q_i(s_i)$ has to equal unity. Therefore the corresponding Lagrange multiplier λ_i is a free or unconstrained variable.

$$\lambda_i \text{ free} \quad , \quad 0 = \sum_{s_i \in S^i} q_i(s_i) - 1 \tag{33}$$

3.2 Implementation in GAMS

Having obtained the system of equations for both games, their implementation in GAMS is straightforward. In the previous section it was already mentioned that the special syntax of specifying MCPs in GAMS has to be considered. Details on implementing mixed complementarity problems in GAMS can be found in [Ferris et al. 2000] or McCarl's GAMS User Guide [McCarl 2004]. Once the implementation is achieved, the problems can be solved by invoking one of the solvers that are suitable for dealing with MCPs, i.e. either MILES [Rutherford 1993] or PATH [Dirkse et al. 1995, Ferris et al. NA].

As briefly mentioned in section 2.3.2, the payoff matrices' size grows exponentially with the number of players. GAMS, not being very efficient in performing simple arithmetic operations, is therefore not particularly suitable to generate the payoff matrices from the results of the supply game. Instead a small C++ program is used to generate the payoff matrices.

3.2.1 Supply Game

In the case of the linear inverse demand function where the equilibrium is unique, there are little problems for the solvers. The choice of an appropriate starting points is of some importance for the solver algorithms though to achieve convergence within the given iteration and time limits. Here PATH usually does a better job, even if no initial starting point is provided by the user. For MILES a well-scaled problem is an important condition for a successful run and it is much more crucial to supply a starting point in the vicinity of the solution to find an equilibrium within the iteration and time limits.

3.2.2 Game of Timing

The situation is more complicated for the game of timing. Apart from the symmetry mentioned in section 2.3.2 there are no other general symmetries, since the payoff matrices are generated dynamically. Furthermore we do not know of any proof of concavity. Hence, depending on the model parameters, multiple equilibria might exist. This topic will be discussed in section 3.3.

It is interesting to note that, contrary to the behavior in the supply game, with very large timing games[2] MILES usually performs better than PATH. In particular this is true if initial values close to a solution are provided. MILES very quickly finds the solution in such a case, whereas PATH might run into iteration or time limits. Therefore it is advisable to use both solvers, especially when one has to deal with large problems.

For large timing games internal GAMS limitations require the system of equations to be restructured. An upper limit on the nonlinear code block size restricts the sum over s_{-i} in eq. (32)[3]. Thus one has to split the sum into smaller parts, introducing auxiliary variables that finally have to be summed up in another equation. However, this trick has the side effect of slowing down the solution algorithm considerably. Moreover the splitting of equations only postpones the problem to some extent, because of the payoff matrices' exponential growth with respect to the number of players.

3.3 Multiple Equilibria

In contrast to the supply game where concavity of the payoff functions $\pi_i(y_i, y_{-i})$ ensures that a unique equilibrium exists [Rosen 1965, Gabay et al. 1980], we do not know of a similar proof for the game of timing. Hence, in general we have to assume that multiple equilibria in pure as well as mixed strategies exist. Different concepts try to classify and reduce the number of equilibria. In the context of games in normal form, some of them are *proper, persistent* and *stable* equilibria. Special algorithms are required to implement these concepts into models.

[2]Games with 4 or 5 players, each having between 10 and 20 strategies are considered to be large. This corresponds to payoff matrices with $10^5 - 10^6$ elements.

[3]GAMS terminates the execution with the error message *Single block NL code size exceeded.*

For an overview over the advanced concepts of Nash equilibria see e.g. [Kohlberg et al. 1986].

In this paper we rely on the standard Nash equilibrium and try to identify the equilibria by (i) employing two different solvers to find solutions of the MCP and (ii) performing multiple solver runs with randomly generated starting points (see e.g. [Zhigljavsky 1991, chp.2.1]). Obviously this rather simple approach does not ensure that we will find all existing solutions of the MCP, but with an increasing number of model runs, the chance to find existing solutions improves. The Nash equilibria can then be compared by taking into account other criteria, e.g. Pareto efficiency.

Usually, the larger the number of players and strategies the more equilibria exist. On the other hand, we find that if the economic and technical parameters of the competing projects differ sufficiently from each other, the number of Nash equilibria in pure strategies, identified by the solvers is very limited. Apparently, mixed strategy equilibria play a larger role if the parameters are very similar for the projects. In particular the symmetric situation exhibits multiple Nash equilibria in mixed strategies. In section 3 of [Rutherford 1995] examples for the efficiency of locating Nash equilibria in finite n person games are given. [McKelvey et al. 1996] provides an overview over the computational problems involved in the treatment of this class of games.

4 Sensitivity Analysis

Most parameters involved in this analysis are estimates of future developments and significant uncertainties have to be associated with them. Therefore we will perform a detailed sensitivity analysis, apart from parametric uncertainties also including a variation of important model assumptions. Among these the inverse demand function (4) as well as the objective function (15) certainly play an outstanding role and hence will be considered in the following sections.

4.1 Parametric Uncertainties

4.1.1 Supply Game

For the treatment of parametric uncertainties in the supply game we will rely on the method proposed by Rutherford [Rutherford 1995] which is particularly suitable for the implementation we have chosen. Suppose a system of nonlinear equations

$$F(y, \bar{x}) = 0 \tag{34}$$

has been solved as a MCP. Here y is the vector of variables and $\bar{x}$ are the mean values of the uncertain parameters.

Given a solution $y^\star$, the influence of variations of the uncertain parameters x around their mean values $\bar{x}$ on the solution provides a measure of the solution's robustness. The total derivative of $F(y, \bar{x})$ taken at the solution $y^\star$ is

$$dF|_{y^\star} = \nabla_y F dy + \nabla_x F dx = 0\,, \tag{35}$$

where $\nabla_y F$ and $\nabla_x F$ denote the Jacobians of F with respect to y and x respectively. At the solution $y^\star$ the derivative has to vanish by definition (first-order condition). Solving for dy reveals

$$dy = -(\nabla_y F)^{-1} \nabla_x F dx = S dx\,, \tag{36}$$

where S is called *local sensitivity matrix*. According to [Rutherford 1995] S can be calculated by solving the following set of nonlinear programs for every variable y_i.

$$\begin{aligned} &\max_{y,x}\, y_i && , \\ &\text{s.t. } F(y,x) = 0 && , \quad x = \bar{x} \end{aligned} \tag{37}$$

For y_i being the objective, the dual activities associated with the restrictions on x correspond to the ith row of the sensitivity matrix S. Hence the matrix can be calculated by solving n separate nonlinear programs.

In the case of a MCP, the nonlinear program, i.e. $F(y, x)$ consists only of those equations that are binding at the solution $y^\star$. Equations with marginal values equal to zero thus have to be omitted from the system of equations F. As a result uncertainties in restrictions that are non-binding at the solution have no impact on the solution's uncertainty. For instance if a pipeline does not work at full capacity at the solution, the uncertainty associated with the capacity parameter does not contribute to the uncertainties of the price and supply levels. This is even true if the supply level is only a tiny fraction from the capacity constraint away. For that reason this method is *local* sensitivity analysis.

When determining the impact on the players' profits, it has to be kept in mind that the price is a function of various parameters (p_0, e_p) as well as supply levels. Hence the results for the uncertainty of price and supply levels, obtained from the local sensitivity analysis, cannot be varied independently to calculate the uncertainties in the profit functions. This dependency is also of importance when deriving the uncertainties of the payoff-matrix elements, which are an input to the game of timing. To explicitly calculate the influence of uncertainties in all input quantities (results of supply game, investment cost, lifetime, discount rate, ...), we employ a simple Gaussian error propagation for statistically independent parameters.

In the analysis uncertainties in the reference gas price p_0, the absolute value of the price elasticity e_p, the upper price limit $p_{\max}$, the variable extraction and transport costs c_i^{ext}, c_i^{var} and the fixed and annualized investment costs c_i^{fix} and c_i^{ann} are taken into account. Some of the parameters influence the projects' profits π_i only through their impact on the natural gas price p and the supplies y_i whereas others directly appear as coefficients in eq. (5). The approximate uncertainty of π_i can be expressed as

$$\begin{aligned}
\Delta\pi_i^2 &= +\left(\frac{\partial\pi_i}{\partial p_0}\Delta p_0\right)^2 + \left(\frac{\partial\pi_i}{\partial e_p}\Delta e_p\right)^2 + \left(\frac{\partial\pi_i}{\partial p_{\max}}\Delta p_{\max}\right)^2 \\
&\quad + \left(\frac{\partial\pi_i}{\partial c_i^{\text{ext}}}\Delta c_i^{\text{ext}}\right)^2 + \left(\frac{\partial\pi_i}{\partial c_i^{\text{var}}}\Delta c_i^{\text{var}}\right)^2 + \left(\frac{\partial\pi_i}{\partial c_i^{\text{fix}}}\Delta c_i^{\text{fix}}\right)^2 \\
&\quad + \left(\frac{\partial\pi_i}{\partial c_i^{\text{inv}}}\Delta c_i^{\text{inv}}\right)^2 + \left(\frac{\partial\pi_i}{\partial r_i}\Delta r_i\right)^2 \\
&= \Delta\pi_i^{\prime 2} + \left(\frac{\partial\pi_i}{\partial r_i}\Delta r_i\right)^2 . \qquad (38)
\end{aligned}$$

The first three terms are obtained from the solution of the NLPs, as described above. The two terms in the second row have contributions from the NLPs through the price p and the supplies y_i, but also a direct impact on π_i. Finally the terms from the third row only have direct contributions, since the profit condition is independent of them. The annuity of investments c_i^{ann} is – as can be seen from (6) – a function of investment costs c_i^{inv} and the discount rate r_i. Since the discount rate is also a parameter in the payoff matrices Π_i for the game of timing (15), its impact on the uncertainty of the profit functions π_i cannot be aggregated into a total uncertainty $\Delta\pi_i^{'}$. Hence, we adapt here investment costs and discount rate as independent parameters. All parameters apart from the discount rate r_i do not appear in (15) and thus their contribution to the uncertainty $\Delta\pi_i$ can be summarized in the term $\Delta\pi_i^{'}$. The impact of uncertainty in r_i is treated separately and thus r_i can be varied independently.

4.1.2 Game of Timing

Unfortunately the above described technique is not applicable to the game of timing if the solutions are Nash equilibria in pure strategies. For pure strategies $q_i(s_i) = 0$ or 1 for all s_i, i and therefore all inequalities become binding. Hence the solution is completely fixed and in the corresponding NLP the parameters' marginals are all equal to zero. For mixed strategy equilibria this is not the case, but since we are mostly interested in pure strategy equilibria, an implementation of the technique does not seem to be reasonable.

Consequently, we do the following to assess the impact of parametric uncertainties on the outcome of the timing game. As described in the previous section we estimate the uncertainties of the payoff matrix elements by using the results from the supply game uncertainty analysis. The corresponding Gaussian error propagation formula for the payoff matrices is

$$\begin{aligned} \Delta\Pi_i^2 &= \left(\frac{\partial \Pi_i}{\partial \pi_i}\Delta\pi_i'\right)^2 + \left(\frac{\partial \Pi_i}{\partial r_i}\Delta r_i\right)^2 \\ &= \sum_{t=t^0}^{t^1}\Bigg\{(1-r_i)^{2(t-t^0)}\Delta\pi_i'^2 + \Bigg[(1-r_i)^{(t-t^0)}\cdot\frac{\partial \pi_i}{\partial r_i} \\ &\qquad -(t-t^0)(1-r_i)^{(t-t^0-1)}\cdot\pi_i\Bigg]^2\Delta r_i^2\Bigg\}, \end{aligned} \tag{39}$$

where for the sake of simplicity the Θ- and δ- functions are not shown. Subsequently the payoff matrices are varied within these boundaries, taking into account the correlation of parameters and the influence on the equilibria is observed.

4.2 Alternative Inverse Demand Function

The linear inverse demand function that has been applied so far is a particularly simple and also easy to handle choice. Certainly it is not unique and from the theoretical point of view not the most convincing function. Therefore, as an alternative, we introduce an isoelastic inverse demand function as was used in [Klaassen et al. 2003, Golovina et al. 2002] to observe how important its choice is for the model results. Using the same independent parameters as in eq. (4), we obtain

$$p(y) = p_0\left(\frac{d_0}{y}\right)^{\frac{1}{e_p}}. \tag{40}$$

Apparently $p(y) \to \infty$ as $y \to 0$ which (i) might cause numerical problems and (ii) is unrealistic. To cure the first problem we introduce a small parameter ϵ which is added to y in the denominator of (40) to avoid divergences. The incentive to increase profits by further and further reducing supplies can be removed by imposing an upper price limit as discussed in section 2.2.2. This constraint can then be chosen to equal the price at zero supply for the linear inverse demand function to make both cases more comparable.

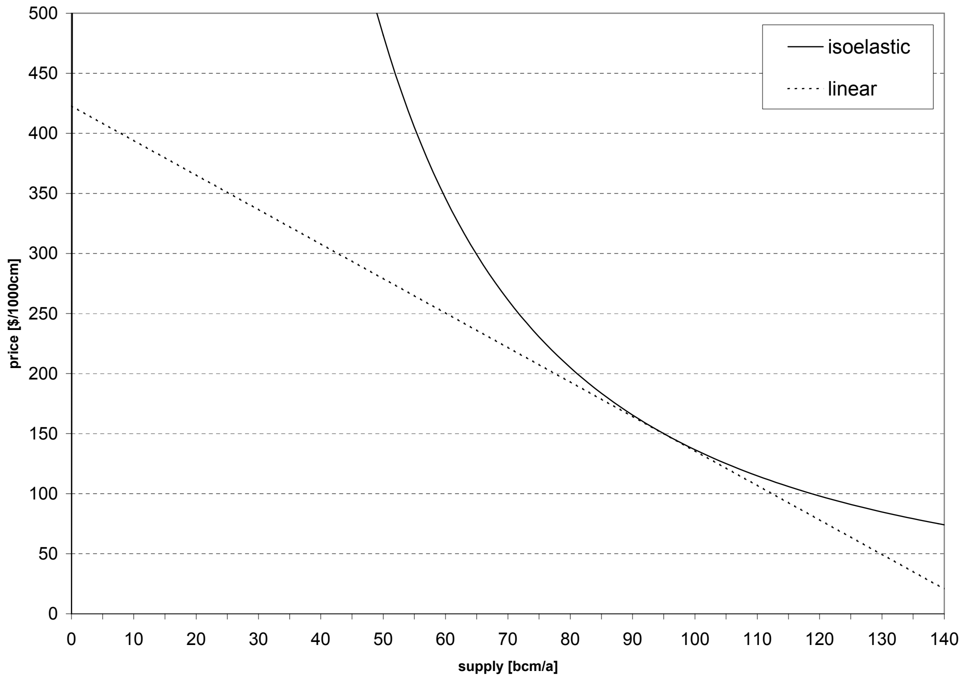

Figure 1: Inverse demand function: linear and isoelastic.

Figure 1 provides an overview over the different shapes of the inverse demand functions, based on the same set of parameters p_0 and e_p.

With the isoelastic inverse demand function, the profit condition (27) is changed to

$$y_i \geq 0 \quad \perp \quad 0 \geq p_0 \cdot \left(\frac{d_0}{y}\right)^{\frac{1}{e_p}} \left(1 - \frac{y_i}{e_p y}\right) - (c_i^{\text{ext}} + c_i^{\text{var}}) - \lambda_i^{\max} + \lambda_p \frac{p_0}{e_p} \cdot \frac{d_0^{\frac{1}{e_p}}}{y^{\frac{1}{e_p}+1}} . \tag{41}$$

The inverse demand function also has an impact on the endogenously calculated price. Hence (29) is replaced by

$$p \text{ free} \quad , \quad p = p_0 \left(\frac{d_0}{y}\right)^{\frac{1}{e_p}} . \tag{42}$$

In addition the upper price limit (31) is modified to be

$$\lambda_p \leq 0 \quad \perp \quad p_0 \left(\frac{d_0}{y} \right)^{\frac{1}{e_p}} \leq p_{\max} \,. \tag{43}$$

5 Application to China's Natural Gas Market

To illustrate the operation of the model we will apply it to the Chinese natural gas market, because of its expected dynamics. An outline of the natural gas market's development is followed by an overview of a number of proposed pipelines that will serve us as players for our model analysis.

5.1 Natural Gas Demand

The future demand for natural gas in China has two driving forces, (i) the need for new sources of energy driven by strong economic growth and (ii) the desire to reduce coal combustion which causes serious pollution, especially in urban areas [IEA 2002a, Logan et al. 2002, Rui 2004]. The balance between these main reasons varies among provinces. In the north with its large coal reserves and correspondingly high utilization of coal, atmospheric pollution is a serious problem. On the other hand the economically booming southern coastal provinces require sufficient energy supplies to ensure continuation of economic growth. The IEA assumes an economic growth of annually 5.7% between 2000 and 2010 and 4.3% for the period from 2010 to 2030 [IEA 2002b].

In the year 2000 coal contributed about 70% to the Chinese primary energy supply (excluding non-commercial energy), whereas the share of natural gas was around 3%, corresponding to about 30 billion cubic meters (bcm) [IEA 2002b]. To support the transition to natural gas Chinese authorities have issued the target to increase domestic production by a factor of 2 between 2001 and 2005 and to double the share of natural gas in the country's primary energy mix within ten years [IEA 2002a]. Along with the exploration of domestic resources, massive investments in the gas infrastructure are planned to be made. It is worth noting that most existing pipelines connect a single gas field to a single consumer (which is in many cases a fertilizer plant). So there is no transportation and distribution infrastructure to supply gas to a large number of industrial or even residential customers. Since 1996 a number of long distance pipelines have already been built [IEA 2002a] and the decision to construct the 4000 km long West-East Pipeline (WEP), which will transport about 12 bcm/a of natural gas from the Tarim Basin to Shanghai, was the centerpiece of the government's plan to setup a national gas transmission system [IEA 2002a, IEA 2003].

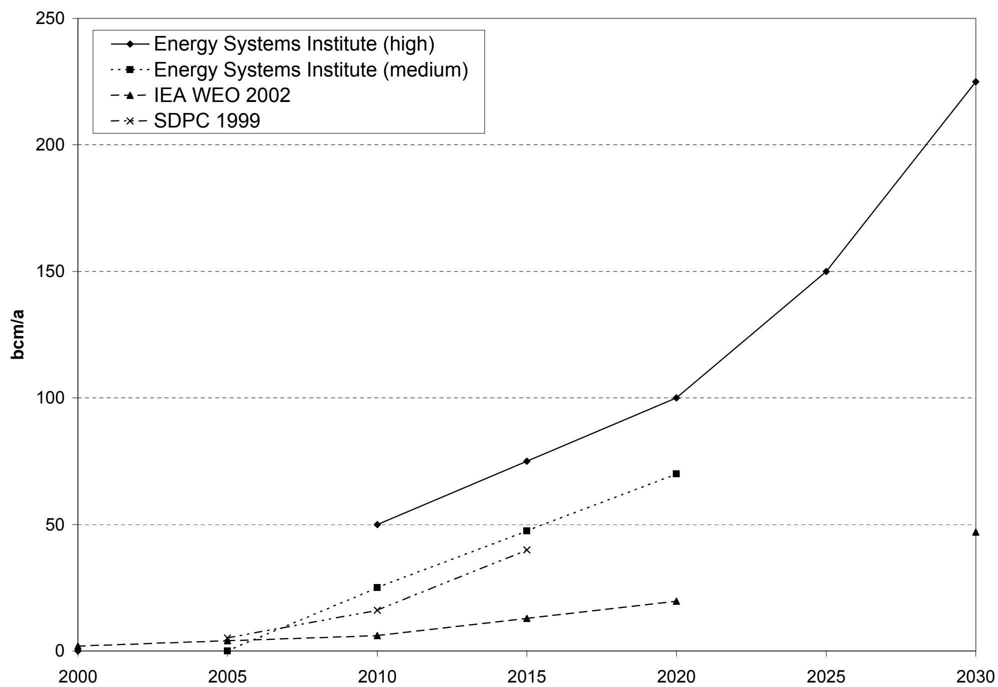

Figure 2: Forecasts of natural gas demand gap in China

Although recoverable domestic resources are considerable (1300 bcm [IEA 2002a], 2100 bcm [Logan et al. 2002]), most likely they will not be sufficient to cover the growing demand for natural gas in the future [IEA 2002a, Brennand 2001]. Corresponding to the differences in the natural gas demand projections, the expected arising demand gap that needs to be covered by imports also varies significantly. The IEA projects a demand gap of 47 bcm by 2030, corresponding to 29% of total gas demand [IEA 2002b]. Other sources, mentioned in [IEA 2002a], assume imports of 44-64 bcm already by the year 2015. In [Logan et al. 2002] a demand gap of 39.9 bcm by 2015, based on the State Development Planning Commission's (SDPC) *baseline demand scenario*, is given. The corresponding *high demand scenario* implies a 22 bcm larger import requirement. Data provided by the Energy Systems Institute (ESI), Irkutsk, Russia [Kononov et al. 2004] indicate a demand gap of 65 bcm in 2015 and finally reach import requirements of 225 bcm in 2030. A more moderate scenario by [Kononov et al. 2004] estimates the demand gap to be around 25 bcm/a in 2010 and 70 bcm/a in 2020. For comparison, trends corresponding to the above described scenarios are presented in Figure 2.

Although the natural gas market was highly regulated in the past, positive trends towards market liberalisation can be observed. For projects developed after 1997 the contract price is freely negotiable between producer and consumer, thus giving grounds to assume the establishment of a market price formation in the future. In reality the contract price still has to be approved by the State Development Planning Commission, but the new price system can be considered as a first step towards deregulation of natural gas prices in China [IEA 2002a, Logan et al. 2002, Locatelli 2004].

5.2 Proposed Pipelines

There are a number of major pipeline projects which have been proposed to deliver natural gas from former CIS countries to the Chinese market. Figure 3 gives an overview over the region and shows the routes of the pipelines. Three projects are designed to deliver gas from East Siberia and the Russian Far East to northeast China. The remaining pipelines are taking routes west of Mongolia and enter China in the northwest region, close to the Tarim basin where the largest Chinese gas reserves are located. The list of pipelines, where the names

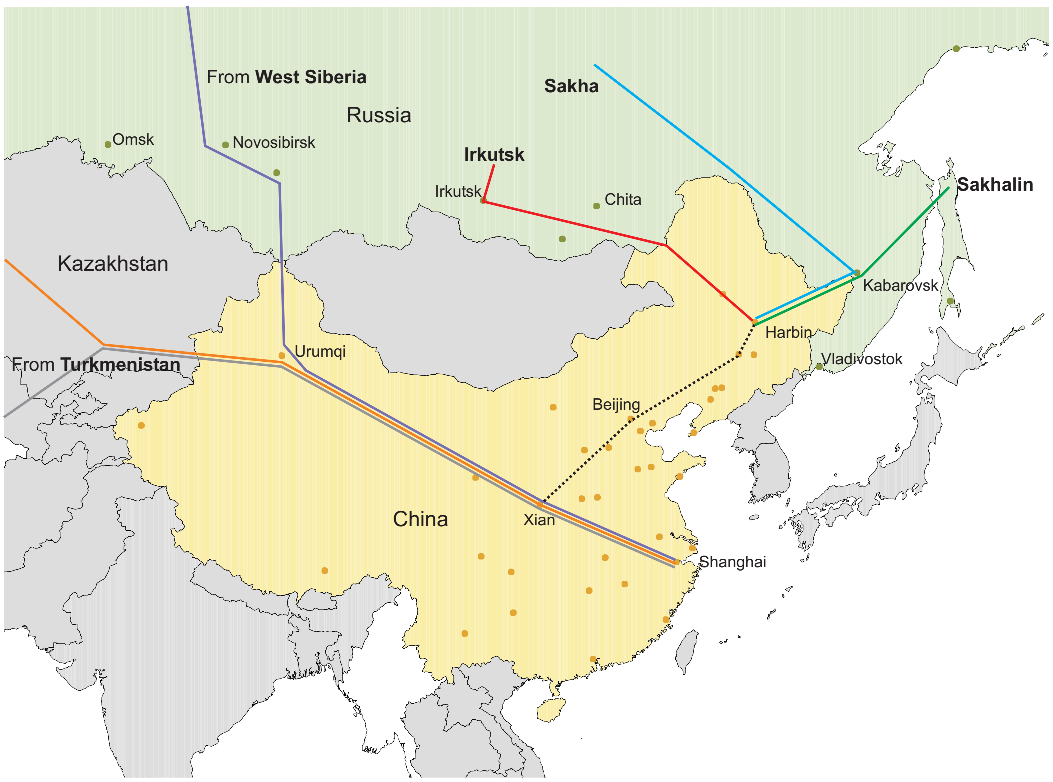

Figure 3: Proposed routes for natural gas trunk lines in Northeast Asia.

in bold characters are used to refer to the pipelines in the following parts of the paper is presented below:

- Kovykta - **Irkutsk** - Zabaikalsk - Harbin

  The most promising project so far is a pipeline from Irkutsk's Kovyktinskoe field to northeast China and further to South Korea. For this project a feasibility study has been completed and approved in November 2003 and preliminary letters of intent for purchasing and selling gas have been signed [CNPC 2003].

- **Sakhalin** - Khabarovsk - Harbin

  This pipeline would deliver gas from the Sakhalin-1 field via Khabarovsk to north-east China. For Russia the project would include the possibility to develop its gas infrastructure in the Khabarovsk and Vladivostok regions where the demand could also grow rapidly [Harrison 2002].

- Republic **Sakha** - Khabarovsk - Harbin

  This pipeline could be realized either as a single project or as an extension of the Irkutsk pipeline to ensure long-term supply from East Siberia to China and South Korea. Development costs for this project are estimated to be higher than for the Kovykta gas field, due to harder natural conditions (e.g. permafrost) and the dispersed locations of gas fields over a large region [IEA 2002a].

- **West Siberia** - Gorno-altaisk - Shanshan - Shanghai

  In 1997 an agreement between CNPC and Gazprom was concluded to supply gas from West Siberia through the Altai mountains to the Tarim Basin and further to Shanghai or Beijing [IEA 2002a].

- Karachaganak gas field (**Kazakhstan**) - Shanshan - Shanghai

  There are activities of China's CNPC in the development of oil fields in Kazakhstan and the construction of an oil pipeline is projected. Natural gas supply to China from the Karachaganak gas field to the Tarim Basin and then to east China has been included into a scenario by the Asia-Pacific Energy Research Centre (APERC) [APERC 2000].

- **Turkmenistan** - Shanshan - Shanghai

The possibility to deliver natural gas from Turkmenistan to China and Japan has been the subject of an agreement between these three countries. A feasibility study for the pipeline from Turkmenistan through Uzbekistan and Kazakhstan to China has been carried out in the 1990's. Due to its poor economic evaluation and the risks of political instability, the project is the least probable to be realized [IEA 2002a].

In the IEA World Energy Investment Outlook it is assumed that the first two projects are going to be built in the decade after 2010, whereas the projects from Kazakhstan and Turkmenistan are expected to start operation not before 2020 [IEA 2003].

6 Calculations and Results

Based on the situation in China and Northeast Asia, described in the previous section, a scenario for the game theoretic analysis is selected. The required input data for the model will be presented, followed by model results and a sensitivity analysis.

6.1 Input Data

6.1.1 Pipeline Projects

In section 5 we have given an overview of a number of pipeline projects that have been proposed to supply natural gas to the Chinese market. As was mentioned, a feasibility study for the pipeline from Turkmenistan to China came to the conclusion that the project's economics was rather poor. Hence we will exclude this project and only consider the remaining five pipelines in the following analysis. Concerning the other projects it has to be mentioned that the data only represent the share of the project that is designated to deliver natural gas to China. For instance the trunk of the Irkutsk pipeline has a designed capacity of 30 bcm, of which 10 bcm are to be delivered to South Korea. In the present analysis only the Chinese market is considered though and hence the data represent two thirds of the physical pipeline. Economic and technical data of all projects are summarized in Table 2.

	units	Irkutsk	Sakhalin	Sakha	West Siberia	Kazakhstan
earliest start	[-]	2010	2010	2015	2019	2019
length	[km]	2800	1800	4200	6100	5100
capacity	[bcm/a]	20	12.5	17.5	30	25
inv. cost	[mill.$]	6500	3500	8800	15000	12000
fix. cost	[mill.$/(bcm/a)]	4.9	4.2	7.5	7.5	7.2
ext. cost	[$/1000cm]	47.5	55	60	40	30
var. cost	[$/1000cm/ 1000km]	22.5	25.5	26.5	22	23

Table 2: Economic and technical parameters of pipeline projects

Most data have been provided by the ESI, Irkutsk, Russia [Kononov et al. 2004] and have been reconfirmed by comparing them with data from other sources [NAGPF 2004, Troner 2000, Paik 2005, IEA 1994, IIASA ECS Program 2003]. Construction costs for the projects found in the literature and various press releases show a considerable spread (e.g. [Pipeline & Gas Journal 2004]). This is mainly explained by the fact that (i) these data are a commercial secret and (ii) most projects are still in an early planning stage. For some of the the projects even the pipeline route is not finally settled, consequently resulting in different cost estimates. The specific variable transport costs in Table 2 fluctuate around 25$/(1000cm 1000km) which corresponds to values from other sources (e.g. [IIASA ECS Program 2003]). Extraction or wellhead costs strongly depend on the conditions under which gas is extracted. The bandwidths in [Troner 2000] are comparable to the data in Table 2. Due to the lack of data for fixed operation and maintenance costs, we follow a common practice in the literature and many model applications and set them equal to a fraction of the total investment costs. As a standard value 1.5% of the investment costs is used, deviations from this value have been considered in the sensitivity analysis.

For all projects we assume technical and economic lifetimes of $t_i^{\text{tech}} = t_i^{\text{eco}} = 30$ years. Also the discount rates are chosen to be equal at $r_i = 10\%$. The latest possible year of entering the market is $t_i^1 = 2020$ for the Irkutsk and Sakhalin pipelines and $t_i^1 = 2030$ for the others.

6.1.2 Market Data

For the description of the market behavior, the parameters p_0 and e_p for a given reference demand d_0 are required to determine the inverse demand function. As mentioned before, the existing forecasts for the demand gap show considerable variations. Correspondingly the impact on the parameters of the inverse demand function is rather large. However in our analysis we choose only one scenario, mainly to demonstrate the model's applicability. The scenario is based on the forecasts by the Energy Systems Institute, Irkutsk (Russia) and will be referred to as the ESI scenario. The estimates for the price p_0 and e_p shown in Table 3 are provided by the ESI [Kononov et al. 2004] and [Kononov 2001]. The price elasticity is

parameter		units	2000	2010	2020	2030
ESI demand	d_0	[bcm/a]		95	175	255
domestic supply	y_{ext}	[bcm/a]		70	105	140
demand gap	$d_0 - y_{\text{ext}}$	[bcm/a]		25	70	115
reference price	p_0	[$/1000cm]	135	150	165	165
price elasticity	e_p	[-]	0.55	0.55	0.50	0.45
price limit	$p_{\max}$	[$/1000cm]	-	-	-	-

Table 3: Inverse demand function parameters.

an average of the values for power plants and industrial sector. For the time after 2030 all parameters are fixed to their 2030 values, i.e. we do not extrapolate trends. This approach is discussed in the context of the model's time horizon in the following paragraph.

6.1.3 Time Horizon

The choice of the model's time horizon is of particular importance for the model results. Apparently the further the time horizon is extended, the larger the uncertainties become. Also it is difficult to find estimates of demand and prices, that are required to fix the parameters in the inverse demand function, beyond 2030 in the literature.

Apart from these difficulties there are model inherent problems: Data are only available for a limited number of projects. Furthermore the projects' technical lifetime is fixed to 30 years and hence pipelines start to phase out from 2040, assuming first possible market entry in 2010. Obviously the resulting reduction of total supply leads to an increasing price, because there are no additional pipeline projects that could step in to fill the gap. This again can create incentives for other projects to postpone their market entry if the increase in profits compensates the depreciation due to discounting. Therefore it is not advisable to run the model way beyond the time of first possible phase out, i.e. in our case 2039. Similar effects occur if demand data are simply extrapolated for the time periods beyond 2030, which might lead to very high natural gas prices. Thus we choose for the base year t_0 and the cut off time t_1 the following values:

$$t_0 = 2000 \quad \text{and} \quad t_1 = 2039$$

The use of the annuity of investments in eq. (5) guarantees that the current market value of the pipeline at t_1 is added to the projects' net present value to compensate for the operation time cut off. Thus only a share of the investment costs has to be recovered, corresponding to the ratio of the project's utilization time to economic lifetime.

6.2 Results

6.2.1 ESI Scenario

For the demand projections of the ESI scenario we find two Nash equilibria in pure strategies[4]. As described in section 3.3, we have run the model with a number of randomly generated initial strategy combinations. The basis of the present analysis are 10000 model runs, where the order of magnitude of possible pure strategy combinations is 10^6.

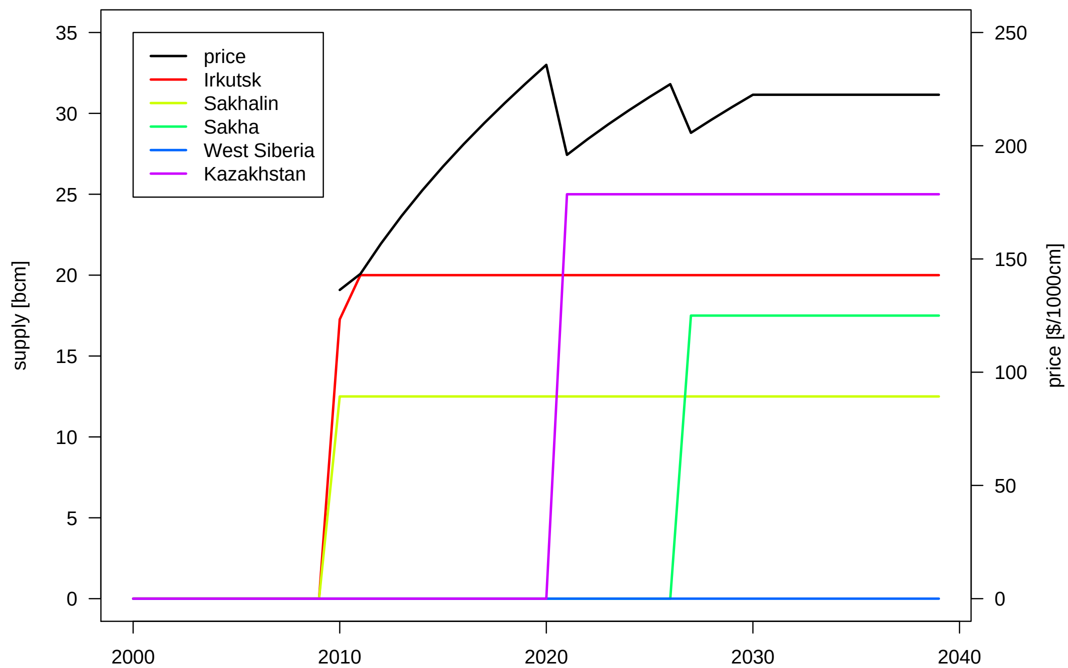

Figure 4: Physical flows and natural gas price for ESI scenario (equilibrium 1).

Figures 4 and 5 show the evolution of natural gas supply by individual pipelines on the left (colored curves) and the corresponding natural gas price on the right axis (black curve). In Figures 6 and 7 the cumulated discounted cash flow for the different projects in the ESI scenario is displayed. For this graph the base year of discounting is the year of project

[4]The number of Nash equilibria in finite games in normal form is odd. Therefore another equilibrium, possibly in mixed strategies, should exist.

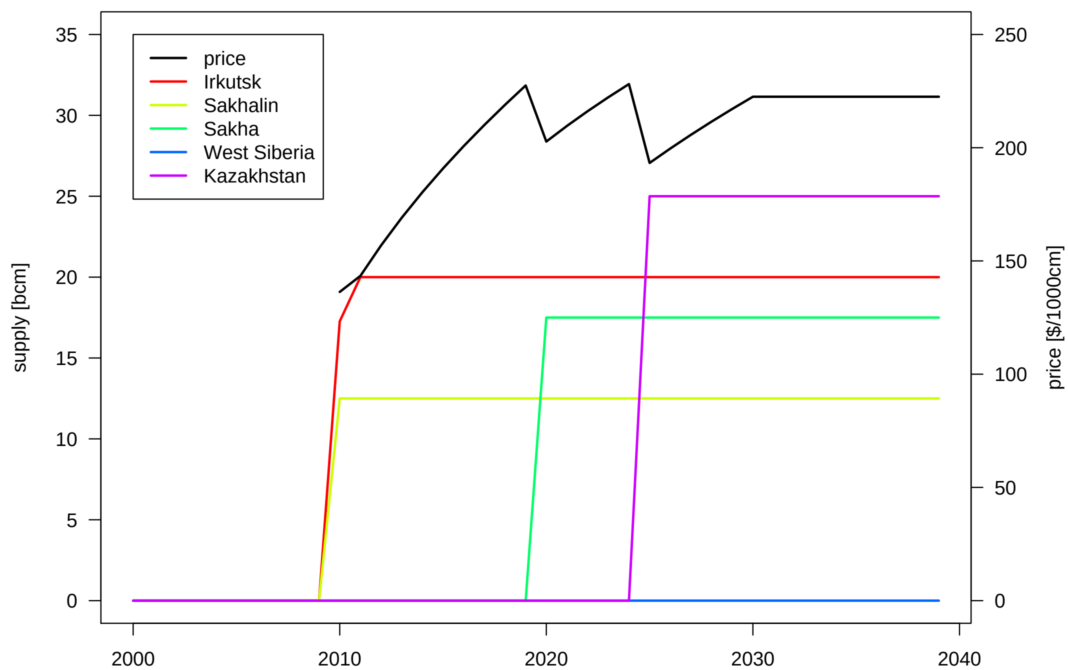

Figure 5: Physical flows and natural gas price for ESI scenario (equilibrium 2).

commercialization, in contrast to the common base year t^0 that is used in the objective function of the game of timing. Also, the full investment costs have been included to generate the graph, not just the share corresponding to the time of operation within the model's time horizon. Therefore the negative peak of the curves corresponds to the total investment costs, which have been uniformly distributed over a three-year construction period for illustrative purposes.

Given the demand projections of ESI, the Irkutsk pipeline in both solutions enters the market at the earliest possible time, but does not run at full capacity during the first year. Also the pipeline from Sakhalin is built as soon as possible and, due to its comparatively small capacity, starts utilizing its full capacity immediately. The Sakha and Kazakhstan pipelines enter the market in interchanged order in the two solutions. In equilibrium 1 (see Figures 4 and 6) the Sakha pipeline enters the market 12 years after the earliest project start (2027).

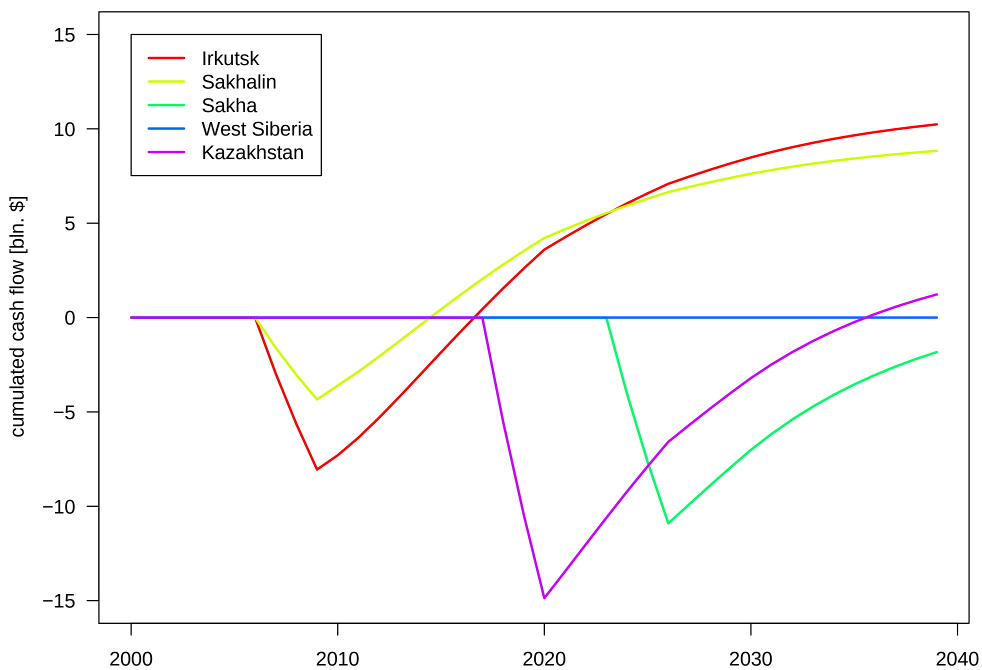

Figure 6: Cumulated cash flow in the ESI scenario (equilibrium 1).

The high extraction costs and the long pipeline length are to be blamed for that. Supplies from Kazakhstan are more competitive and thus this pipeline is realized before the Sakha project, but still one year later than possible (2021). In equilibrium 2 (see Figures 5 and 7) the Sakha project seems to profit from its earlier possible start which results in a delayed start of the Kazakhstan pipeline. Natural gas supplies from Sakha start in 2020, i.e. five years after the earliest possible start and the Kazakhstan pipeline starts operation in 2025. These two pipelines start operating at their full capacity in both equilibria. Gas transport from West Siberia cannot compete with the other sources under the given assumptions. The equilibrium solutions suggest not to build this pipeline connection.

The cumulated cash flow by the end of the model's time horizon, i.e. by 2039, is the largest for the Irkutsk project. This is however mainly due to being one of the projects that is at the end of its lifetime by 2039 and having a larger designed capacity than the Sakhalin pipeline

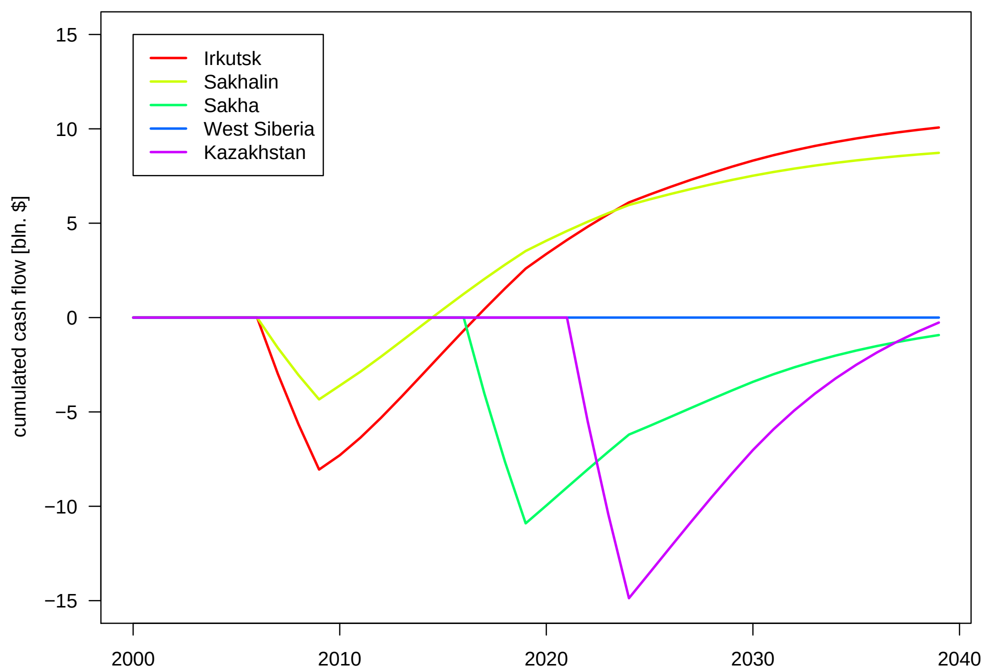

Figure 7: Cumulated cash flow in the ESI scenario (equilibrium 2).

whereas the other two projects still have a number of operation years to await. Comparing the projects' cumulated cash flow at the end of the time horizon for both Nash equilibria, solution 1 is favorable for the Kazakhstan pipeline. In contrast the Sakha pipeline whose start is delayed by six years compared with equilibrium 2, shows a slightly reduced cumulated cash flow by 2039, although the payback time seems to go down in equilibrium 1. For the Irkutsk and Sakhalin pipelines the two equilibria result in an almost identical cumulated cash flow.

Figures 4 and 5 reveal that the market price between 2020 and 2027 is different in the two equilibria, due to the interchanged market entry of the Sakha and Kazakhstan projects. These pieplines have different physical capacities and therefore an impact on the gas price can be observed.

6.3 Sensitivity Analysis

6.3.1 Supply Game

To get an idea of the impact, caused by parametric uncertainties on the results of the supply game, we have varied the concerned parameters independently by 10% and give the resulting influence on price and supply levels in Table 4.

parameter	y_i	y_{-i}	$p(y)$
p_0	$\leq 41\%$	-	$\leq 10\%$
e_p	$\leq 7.5\%$	-	$\leq 4.8\%$
$p_{\max}$	-	-	-
c_i^{var}	$\leq 86\%$	$\leq 29\%$	$\leq 4.4\%$
c_i^{ext}	$\leq 17\%$	$\leq 6.5\%$	$\leq 1.1\%$

Table 4: relative uncertainties due to parametric variations in the supply game results

Generally, the impact of parametric variations on the equilibrium solution strongly depends on the particular game situation (see section 4.1.1). The gas price is least influenced by variations of the parameters. From Table 4 it can be seen that parametric variations on the 10% level can cause variations of up to 10% in the price, but most parametric uncertainties have a smaller impact. It reacts most sensitive to variations of p_0 and e_p which directly appear in the inverse demand function (4). Uncertainties of the projects' variable transport and extraction costs have a smaller impact, because they influence the price $p(y)$ only indirectly via the supplied quantities y. As one can expect, the more players are active on the market the weaker the influence of the individual player's parameters on the price.

The situation is different for the supplied quantities y_i. If a player is close to the threshold of profitability, i.e. if the market price is only slightly above the marginal costs ($c_i^{\mathrm{ext}}+c_i^{\mathrm{var}}$), the relative uncertainty can be large. In the corresponding equilibrium the player's supply is close to zero. Hence parametric variations can cause uncertainties that are large in comparison with the actual supply quantities. Fortunately pipelines with such poor economic performance do not enter the market in the game of timing, because there apart from extraction and variable costs also the investment and fixed costs have to be recovered. Exactly this happens to the West Siberia pipeline, which is responsible for the large relative uncertainties of y_i in Table 4. Therefore the values in Table 4 represent upper boundaries for the relative uncertainty and especially in the case of y_i the average uncertainty is less than 20%.

Since we did not make use of the upper bound on the price, a variation of $p_{\max}$ does not have any impact on the results. Apparently, if the corresponding constraint becomes binding, a variation of $p_{\max}$ is equivalent to a variation of the price p.

6.3.2 Game of Timing

Following the procedure outlined in section 4.1.2, we have varied the players' payoff matrices within the combined uncertainty intervals of all parameters. Hence two additional sets of payoff matrices are generated

$$\Pi_i^{\pm} = \Pi_i \pm \Delta\Pi_i \,. \qquad (44)$$

With these matrices $\Pi_i^{\pm}$ the model was evaluated again, the impact on the Nash equilibria is described in the following paragraph.

Both equilibria show only a moderate response on the variation of the payoff matrices. If Π_i^- is substituted for Π the start of the Sakha pipeline is shifted by one year with respect to the original solution, in equilibrium 1 from 2027 to 2028 and in equilibrium 2 from 2020 to 2021. A similar effect is observed for the Kazakhstan project whose equilibrium strategy stays unchanged for equilibrium 1, but is shifted from 2025 to 2026 for equilibrium 2. The Irkutsk and Sakhalin projects still both enter the market in 2011 in both equilibria, i.e. one later than in the original scenario. A substitution of Π_i^+ shows very similar effects. In equilibrium

1 both the Sakha and Kazakhstan pipelines start to deliver gas one year earlier than in the original scenario, i.e. in 2026 and 2019 respectively. The strategy of the Sakha project remains unchanged (2020) in equilibrium 2, whereas the Kazakhstan project is realized one year earlier (2024).

6.3.3 Alternative Inverse Demand Function

Applying an isoelastic instead of a linear inverse demand function as described in section 4.2 has a moderate impact on the observed equilibria. As can be seen from Figure 1, with identical parameters, the isoelastic inverse demand function yields a higher gas price than the linear function. Only at a total supply y equal to the reference demand d_0 the values of both functions are identical.

Again two equilibria can be found which are closely related to the ones described in the previous section. The result of the different inverse demand functions is a slight shift to earlier market entry for the Sakha and Kazakhstan pipelines. In equilibrium 1 these two pipelines enter the market one year earlier, i.e. in 2020 (Kazakhstan) and 2026 (Sakha). Equilibrium 2 is identical to the second equilibrium in the linear case, i.e. the order of market entry is changed and Sakha enters in 2020 and Kazakhstan in 2025. Another less important modification is the slightly better utilization of the Irkutsk pipeline in the first year of operation, because of the higher gas price.

The price difference between the two model runs with different demand functions is moderate, remaining at less than 10% even in the time period after 2030. Correspondingly the cumulated cash flow is larger in case of the isoelastic inverse demand function for all pipelines. For individual projects the difference can be considerable, e.g. for the Irkutsk pipeline it might sum up to roughly 1.5 bln. $ at the end of the model horizon. However, it is important to note that this has no effect on the Nash equilibria.

6.4 Discussion

In general the equilibria found by the solvers are very robust and show clear preferences for the start of pipeline construction and operation. Neither parametric uncertainties on a 10% level have a large impact on the observed equilibria, nor the variation of the inverse demand function. A shift of one year in the equilibrium solutions is the maximum deviation from the original scenario's outcome.

Almost all pipelines from the start of operation run at their full capacity and thus recover their investment costs rather quickly. This is due to the assumed inverse demand function. Choosing for instance the demand gap projection of the IEA in Figure 2 and a low value for y_{ext} leads to a different picture for all pipelines. After the start of operation they only run at a fraction of the design capacity and only reach their full capacity after 10 to 15 years.

An interesting feature of the observed Nash equilibria is that they all show a natural gas price between 150 and 250 $/1000cm, which is a result of the chosen parameters of the projects as well as of the inverse demand function. This price level is relatively high in comparison with natural gas prices observed so far in the region. Additionally, in China natural gas has to compete with comparatively cheaper coal which is expected to keep its dominating position over the next few decades [IEA 2002a]. However, the model price corridor of 150 – 250 $/1000cm corresponds to estimates for the supply side in [NAGPF 2004, chp.4], being 150 – 200 $/1000cm in 2020. The price level that the demand-side would be willing to pay is considerably lower at 107 – 133 $/1000cm in Bejing [NAGPF 2004, chp.5]. These estimates are based on a very moderate oil price scenario (22 – 28 $/bbl in 2020). Taking the present oil price level around 60 $/bbl into account, the model endogenous price level does not appear to be extremely high. Under such conditions piped gas from Russia and Kazakhstan could be competitive with other energy carriers.

7 Conclusion

We have described a game-theoretic model in which the players' strategies are the times of market entry to analyze the competition between large-scale infrastructure projects. A two-step procedure for the problem's formulation is used, leading to a Nash-Cournot game, the *instantaneous supply game*, and the *timing game* in normal form with discrete times as strategies. Both games are implemented as mixed complementarity problems in GAMS and solved with the solvers MILES and PATH.

The model has been applied to natural gas pipeline projects from former CIS countries to China. In practical applications the competition between up to five players has been analyzed. Special attention was paid to the sensitivity analysis, because most input parameter's (e.g. investment costs and market data) are uncertain to a large extend. Apart from the consideration of parametric uncertainties the analysis was repeated with an alternative inverse demand function. The observed Nash equilibria turned out to be very stable under perturbations of the original data set and the inverse demand function and thus give hints on the competitiveness of the five pipeline projects.

As has been mentioned in section 5, the natural gas market in China is still in transition from a highly regulated to an open market, where the price is dependent on supply and demand only. Therefore the inclusion of a regulating authority as an additional player would be an interesting feature to study. However, the addition of players with different interests (e.g. regulating authority, government) is not straightforward, because an objective function for these players has to be determined. Apparently many possible objective functions for such institutional players exist. A simple implementation of a regulating authority's influence could be the usage of an upper bound on the price. As a matter of fact, the game situation then is reduced to a simple optimization problem if this price restriction is binding. The outcome of the supply game is trivial in such a case, because a player with variable costs below the price limit always operates at full capacity whereas a player with variable costs above the limit does not supply gas to the market. This situation might be interesting only if the price constraint is binding in some years, but not in every year, which requires a very delicately chosen price limit path.

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